

Final State Interactions in Hadronic D decays

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Abstract

We show that the large corrections due to final state interactions (FSI) in the $D^+ \rightarrow \pi^- \pi^+ \pi^+$, $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$, and $D^+ \rightarrow K^- \pi^+ \pi^+$ decays can be accounted for by invoking scattering amplitudes in agreement with those derived from phase shifts studies. In this way, broad/overlapping resonances in S-waves are properly treated and the phase motions of the transition amplitudes are driven by the scattering matrix elements determined in many other experiments. This is an important step forward in resolving the puzzle of the FSI in these decays. We also discuss why the σ and κ resonances, hardly visible in scattering experiments, are much more prominent and clearly visible in these decays without destroying the agreement with the experimental $\pi\pi$ and $K\pi$ low energy S-wave phase shifts.

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1 Introduction

During the last few years a series of theoretical works that combine the Chiral Perturbation Theory (CHPT) expansion [1, 2] with unitarity in a consistent way [3, 4, 5, 6, 7, 8, 9], order by order, have showed up the close relationship between the leading order CHPT amplitudes (driven by the chiral structure and the actual value of the order parameter for the spontaneous breakdown of chiral symmetry, f_π) and the presence, mass and width, together with other properties, of the lightest scalar resonances. These results, concerning the σ resonance, have been also confirmed by the solution of the Roy equations [10].

Also recently, but from an experimental point of view, charmed decays have revealed as a powerful source for the study of the properties of the scalar resonances offering experiments with high statistics, which is of foremost importance taking into account the traditionally lack of high statistics experiments in the scalar sector. In this respect, the studies of the Fermilab E791 Collaboration concerning the decays $D^+ \rightarrow \pi^- \pi^+ \pi^+$ [11], 1686 events, and $D^+ \rightarrow K^- \pi^+ \pi^+$ [12], 15090 events, were the first to point out statistically significant contributions of the σ and κ resonances, respectively. Other Collaborations on D decays also report with high statistical significance the existence of a σ resonance from the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decays, like the CLEO Col. [13], 5299 ± 73 events, Belle Col. [14], 57800 events, and the BaBar Col. [15], 81396 events. Thus, we can conclude that nowadays, with the new data from charmed decays, the existence of the σ should be taken for grant.

Despite these positive facts concerning the charmed decays, from the theoretical point of view several questions arise regarding the analyses followed in the previous references, particularly in the S-waves. We concentrate in this work in the results of the E791 Collaboration with respect to the $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ [16], $D^+ \rightarrow \pi^- \pi^+ \pi^+$ [11] and $D^+ \rightarrow K^- \pi^+ \pi^+$ [12] decays. Although the final state is a three body one, we would expect that, at least, at the low energy tail of the invariant mass of the neutral $\pi^+ \pi^-$ subsystem, where the σ is observed, the interactions of this two pion subsystem with the other pion should be quite soft due to the largeness of the D^+ mass. In this way, the movement of the phase of the σ resonance across the Dalitz plot should be given, according to Watson final state theorem, by the isospin (I) $1/2$ S-wave $\pi\pi$ phase shifts. However, the Breit-Wigner (BW) employed for the σ meson in ref.[11], as it is well known, does not fulfill this property, as we discuss below in more detail. Analogous comments are also appropriate for the κ resonance in the $K^- \pi^+ \pi^+$ decay [12] for the $K^- \pi^+$ subsystem and the $K\pi$ $I = 1/2$ S-wave phase shifts. In addition, controversial properties were also reported by this collaboration regarding the $f_0(980)$ and the $K_0^*(1430)$ resonances. We will develop in this work alternative parameterizations, based on CHPT plus unitarity (chiral unitary approach [17, 18, 19]), to those employed by the E791 Collaboration that do not show the mentioned problems with the S-waves but also contain the σ and κ resonance poles. We will also discuss, based on the presence of Adler zeros, why large destructive non-resonant contributions (called backgrounds by theorists) that completely distort the peak shapes of the σ and κ resonances are present in $\pi\pi$ and $K\pi$ S-wave scattering at low energies, respectively. We will also offer a reason why these backgrounds are not present in charmed decays (also applicable to B decays) due to the lack of such zeroes occurring

only at a specific energy. As a result, we will put on sounder theoretical grounds the important findings of the E791 Collaboration about the relevant role played by the σ and κ mesons.^{#2} Let us stress that in this work we focus on the solution of the previous problems associated with the S-waves but we do not attempt to account for full three body unitarity, this is beyond the scope of this work.

The content of this paper is as follows. After this brief introduction, we consider the $D^+ \rightarrow \pi^- \pi^+ \pi^+$, $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ and $D^+ \rightarrow K^- \pi^+ \pi^+$ decays in sections 2, 3 and 4, respectively. We review the formalism and main findings of refs.[11, 16, 12] regarding these decays, in order, and develop our formalism to take into account FSI in section 2 and apply it to the previous three decays. We also discuss why one should expect clear, although broad, peak structures associated with the σ and κ resonances in these decays. We end with section 5 pointing out the most relevant results of our study.

2 FSI in the $D^+ \rightarrow \pi^- \pi^+ \pi^+$ Decay

The study of the Dalitz plot of the decay $D^+ \rightarrow \pi^- \pi^+ \pi^+$, with a sample of 1686 candidates, was performed in ref.[11] within the Fermilab E791 Collaboration. The estimated signal to background ratio is 2:1, hence we are left with 1124 events and in the study of this decay we will normalize our results to this number of events. Previous studies with lower statistics can be also found in this reference from the E687 Collaboration. In order to introduce the controversial situation regarding the amplitude employed for describing data in ref.[11], let us briefly discussed the fitting process followed by ref.[11]. In this reference, when the decay amplitude is modeled as the coherent sum of well established resonances [20], following the isobar model, the non-resonant term is dominant while the description of the fit is poor. Indeed the χ^2 per degree of freedom, χ^2/ν , turns out to be 254 for 162 degrees of freedom [11]. The main discrepancies between data and the parametrized amplitude come, particularly, from the low energy region, below 0.5 GeV². In order to improve the quality of the fit of the Dalitz plot the authors of ref.[11] included another $\pi^+ \pi^-$ resonance corresponding to the σ resonance whose mass and width were allowed to float in the fit. The quality of the fit substantially improves with a $\chi^2/\nu = 138/162$ and finds values of 478 ± 24 and 324 ± 42 MeV for the mass and width of the σ , respectively. However, the phase of the Breit-Wigner (BW) employed to describe the σ resonance does not follow the elastic S-wave $I = 0$ $\pi\pi$ phase shifts. This is shown in Fig.1a by the dashed line in comparison with the data points corresponding to the elastic S-wave $I = 0$ $\pi\pi$ phase shifts from several experimental references. Indeed, the errors show the variation in the values from one reference to another, see ref.[4] for more details. The discrepancy is manifest, specially close to threshold where the variation of the phase for the BW is much faster than that corresponding to data, despite that at the shown energy the $\pi\pi$ interaction is elastic since the $K\bar{K}$ is above 1 GeV and the 4π only starts contributing in a significant way above around 1.3-1.4 GeV.

Denoting by 1 the π^- and by 2 and 3 the equal positively charged pions, the amplitude

^{#2}Also called $f_0(600)$ and $K_0^*(800)$ in the PDG [20], respectively.

employed in ref.[11] can be written as:

$$\mathcal{A} = a_0 e^{i\delta_0} \mathcal{N}_0 + \sum_{n=1}^N a_n e^{i\delta_n} \mathcal{A}_n(s_{12}, s_{13}) \mathcal{N}_n . \quad (2.1)$$

We describe in detail the ingredients present in the previous equation since we found several errata and omissions in ref.[11] that are necessary in order to reproduce their amplitudes. In Eq.(2.1) the first term is the non-resonant contribution and the other ones originate from the exchange of resonances. Every resonant contribution is Bose symmetrized for the equally charged pions, $\mathcal{A}_n = \mathcal{A}_n[(12)3] + \mathcal{A}_n[(13)2]$, as usual. The parenthesis around 12 mean that the particles 1 and 2 form the resonant state, and analogously for (13). The coefficients a_n and δ_n , for $n \geq 0$, are real constants that float in the fit, the a_n are magnitudes and the δ_n phases. Finally, the \mathcal{N}_n , $n \geq 0$, are normalization factors^{#3} given by:

$$\mathcal{N}_n = 1 / \left(\int ds_{12} ds_{13} |\mathcal{A}_n(s_{12}, s_{13})|^2 \right)^{1/2} . \quad (2.2)$$

The $\mathcal{A}_n[(12)3](s_{12}, s_{13})$ amplitudes for $n > 1$ correspond to the product

$$\mathcal{A}_n[(12)3](s_{12}, s_{13}) = BW_n(s_{12}) F_D^{(J)}(s_{12}) F_n^{(J)}(s_{12}) \mathcal{M}_n^{(J)}[(12)3] . \quad (2.3)$$

The Breit-Wigner propagator is given by,

$$\begin{aligned} BW_n(s_{12}) &= (s_{12} - m_n^2 + im_n \Gamma_n(s_{12}))^{-1} , \\ \Gamma_n(s_{12}) &= \Gamma_n(m_n^2) \frac{m_n}{\sqrt{s_{12}}} \left(\frac{p_1}{\tilde{p}_1} \right)^{2J+1} \frac{F_n^{(J)}(p_1)^2}{F_n^{(J)}(\tilde{p}_1)^2} , \end{aligned} \quad (2.4)$$

with m_n and $\Gamma_n(s_{12})$ the mass and 'energy dependent' width of the (12) resonance and p_1 is the three-momentum of the particle 1 in the (12) rest frame. Furthermore, \tilde{p}_1 is p_1 evaluated at the resonance mass. On the other hand, the $\mathcal{M}_n^{(J)}$ are angular dependent factors and read:

$$\begin{aligned} \mathcal{M}_n^{(0)}[(12)3] &= 1 , \\ \mathcal{M}_n^{(1)}[(12)3] &= -2p_1 p_3 \cos \theta_{13} , \\ \mathcal{M}_n^{(2)}[(12)3] &= \frac{4}{3} (p_1 p_3)^2 (3 \cos^2 \theta_{13} - 1) . \end{aligned} \quad (2.5)$$

where p_1 and p_3 are the three-momenta of particles 1 and 3 and θ_{13} their relative angle, all of them referred to the (12) rest frame. The $F_D^{(J)}$ and $F_n^{(J)}$ are Blatt-Weisskopf penetration factors that depend on the spin J of the resonant state, and are given by:

$$\begin{aligned} F_D^{(0)} &= 1 , \\ F_D^{(1)} &= 1 / \sqrt{1 + (rq_3)^2} , \end{aligned}$$

^{#3}Omitted in the description of the amplitudes given in ref.[11], but used in their results.

$$\begin{aligned}
F_D^{(2)} &= 1/\sqrt{(9 + 3(qp_3)^2 + (rp_1)^4)} , \\
F_n^{(0)} &= 1 , \\
F_n^{(1)} &= 1/\sqrt{(1 + (rp_1)^2)} , \\
F_n^{(2)} &= 1/\sqrt{(9 + 3(rp_1)^2 + (rp_1)^4)} ,
\end{aligned} \tag{2.6}$$

with q_3 the three-momentum of particle 3 in the D^+ rest frame.

The set of resonances that exchange in Eq.(2.1) contain the $\rho^0(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1370)$, $\rho^0(1450)$ and, in the parameterization that reproduces faithfully the experimental data, the σ . As discussed above the most controversial aspects present in Eq.(2.1) involves the $I = 0$ S-wave $\pi\pi$ partial wave.

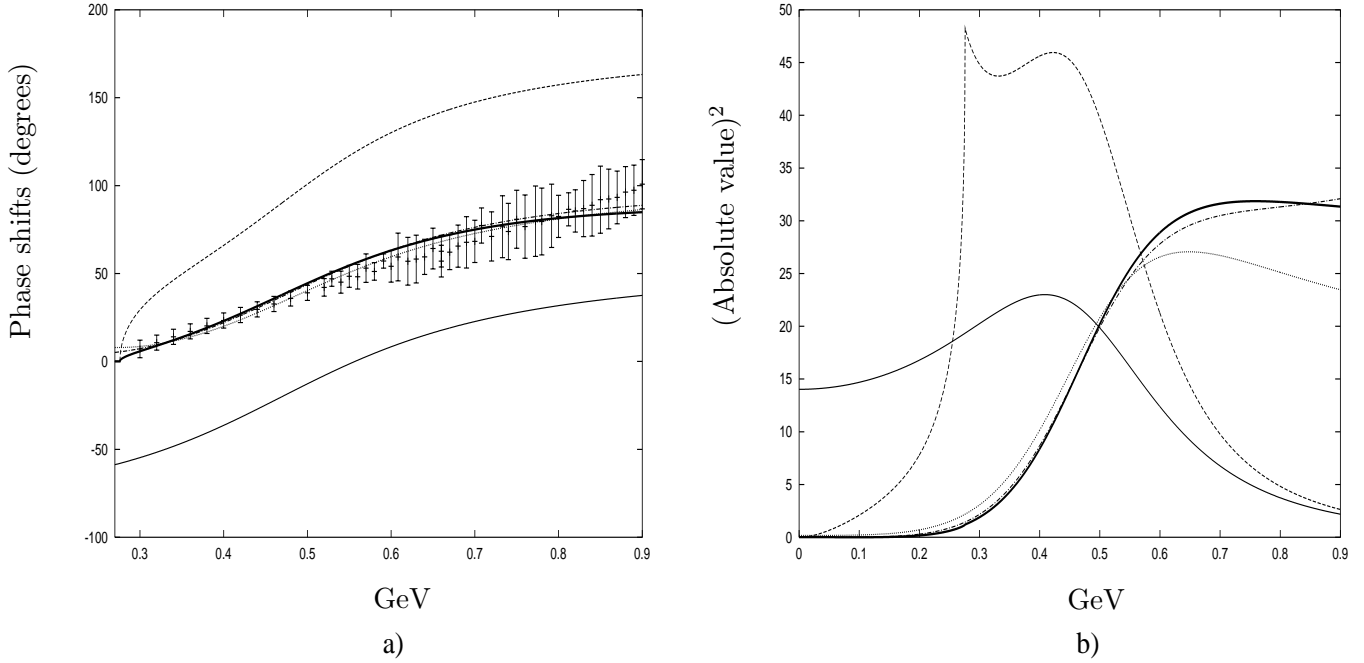


Figure 1: S-wave $I = 0$ $\pi\pi$ phase shifts (left panel) and modulus square of this partial wave (right panel), normalized such that the residue of the σ pole is one. The data points are the elastic S-wave $I = 0$ $\pi\pi$ phase shifts from several experimental references and the errors show the variation in the values from one set to another, as explained in detail in ref.[4]. The dashed lines correspond to the BW of the σ resonance employed by the E791 Collaboration [11]. The thick solid lines are the full results of ref.[3] when keeping only the $\pi\pi$ channel while the thin ones correspond to the pure σ pole contribution of Eq.(2.7). The phase shifts and absolute value from the contribution of the σ pole plus the first non-resonant term in Eq.(4.2) are shown by the dotted lines. Finally, the dashed-dotted lines are the results from the Laurent expansion of Eq.(2.7) keeping all the terms shown.

Now, we want to show that we are able to reproduce the amplitude given in Eq.(2.1) as employed in ref.[11] but using the S-wave $I = 0$ $\pi\pi$, $K\bar{K}$ coupled channel partial waves derived in ref.[3]. These T-matrices were obtained from Chiral Perturbation Theory

(CHPT) at leading order [1, 2] together with a unitarization scheme compatible with the chiral expansion (chiral unitary approach). Furthermore, these matrices are not only able to reproduce the scattering data of the $I = 0, 1$ S-wave amplitudes up to around 1.2 GeV but also have been successfully tested by now in a vast number of production processes that pick up large corrections by FSI from these partial waves, see e.g. [21, 22, 23, 24].

The S-wave $I = 0$ T-matrices from ref.[3] contains two poles in the appropriate Riemann sheets corresponding to the σ and $f_0(980)$ resonances. Their pole positions are around $448 - i224$ and $990 - i13$ MeV, respectively. Let us remark that the given poles are obtained in the full T-matrices derived in ref.[3], which are not any sum of pole contributions. These resonances originate because of the self interactions between the pseudoscalars and are of dynamical origin, disappearing in the large N_c limit [4, 25]. This is why they are said to be of dynamical origin.

Due to the already mentioned discrepancy between phase shifts and the phase motion in energy of the BW for the σ , let us perform a Laurent series around the σ pole position of the $I = 0$ $\pi\pi$ S-wave amplitude,

$$t_{11} = \frac{\gamma_0^2}{s - s_\sigma} + \gamma_1 + \gamma_2(s - s_\sigma) + \dots \quad (2.7)$$

with the pole position and residua obtained from ref.[3] in the elastic case, with the values:

$$\begin{aligned} s_\sigma &= (0.47 - i0.22)^2 \text{ GeV}^2, \quad \gamma_0^2 = 5.3 + i7.7 \text{ GeV}^2, \\ \gamma_1 &= -8.1 + i36.9, \quad \gamma_2 = 1.1 + i0.1 \text{ GeV}^{-2}. \end{aligned} \quad (2.8)$$

In Fig.1 we show in the left and right panels the phase and normalized absolute value of this partial wave, respectively. The thinner solid lines correspond to the pole contribution in Eq.(2.7). We see in Fig.1a that the phase of this pole contribution does not vanish at threshold, but that runs parallel to the experimental phase shifts, so that the difference with respect to them keeps constant along energy up to the increase in the last points due to the closeness of the $f_0(980)$ and the opening of the $K\bar{K}$ channel. Thus, the phase of the σ pole contribution from Eq.(2.7) does follow the motion of the experimental S-wave $I = 0$ $\pi\pi$ phase shifts, in contrast with the BW phase indicated by the dashed line employed in ref.[11]. It is also worth stressing that the phase of the pure σ pole contribution starts at -58 degrees at threshold and this is the reason why its value is not $+90$ degrees at the mass for the σ , 466 MeV, but happens much later, around 1 GeV. By the same reason, for $s \rightarrow \infty$ one gets only $+122$ degrees from this pole contribution instead of the usual $+180$ degrees. The agreement between the experimental phase shifts and the expansion (2.7) is reached rather fast. The dotted lines in Fig.1 correspond to keep as well the γ_1 term in Eq.(2.7), while the dashed-dotted ones correspond to keep all the terms shown in this equation. The thick solid lines are the full results from ref.[3] in the elastic case, removing the $K\bar{K}$ channel. In Fig.1b we consider the absolute value of the partial wave under consideration, but normalized such that the residue at the pole position is one, so that the series in Eq.(2.7) is divided by γ_0^2 . The source of each line is the same as already explained and let us notice that this figure starts at 0 GeV, below threshold. We want to stress three important facts: i) The markedly different behaviour of the σ

Resonance	a_n	δ_n (radians)	Fraction
NR	0.57	0.30	12%
$\sigma\pi^+$	0.10	3.40	79%
$\rho^0(770)\pi^+$	1 (fixed)	0 (fixed)	35%
$f_0(980)\pi^+$	0.47	2.90	8%
$f_0(1370)\pi^+$	0.24	2.09	2%
$f_2(1270)\pi^+$	0.79	1.05	22%
$\rho^0(1450)\pi^+$	0.20	5.30	1%
χ^2/ν	3/152		

Table 1: Results of the reproduction of the parameterization of ref.[11] summarized in Eq.(2.1), removing the BW of the σ for its pole contribution, Eq.(2.9). For every resonance we list the resulting magnitude a_n in the second column, the relative phase δ_n in radians in the third column and the fraction of this decay mode in the fourth one.

BW employed by the E791 Collaboration and the absolute value from the partial wave of ref.[3]. ii) The first peak in the BW. This peaks results because the BW formula (2.4) for the σ resonance generates an unphysical pole in the physical sheet below threshold corresponding to a bound state at around 0.2 GeV. iii) The non-resonant contribution from the terms proportional to γ_1 and γ_2 in Eq.(2.7) are as big as the pole contribution and we see that the final shape of the absolute value of the amplitude is completely distorted as compared with the pure pole contribution. Indeed, the full results from ref.[3] show a zero at around 98 MeV. This is the Adler zero due to chiral symmetry, so that in the chiral limit the pseudoscalar interactions vanish at $s = 0$. At leading order in CHPT [2], this Adler zero sits as well at 98 MeV, very close to the position of the dip in the dotted and dashed-dotted lines and in good agreement with the zero in the total amplitude. In fact, because of the presence of this Adler zero, one can understand why the background turns out to be so big. If there is a pole that affects pretty much the low energy region, as in our case with the σ resonance, it is then necessary a large background to cancel the pole contribution so that an Adler zero can happen.

Before making use of the full results of ref.[3] let us substitute in Eq.(2.1) the BW contribution given in Eq.(2.4), as employed in ref.[11], by the pure σ pole contribution,

$$\frac{a_1 e^{i\delta_1}}{s - s_\sigma}, \quad (2.9)$$

located in the position given in Eq.(2.8). This pole contribution, as discussed above and shown in Fig.1a, has a phase motion in agreement with that from the S-wave $I = 0$ phase shifts. We keep the rest of terms in Eq.(2.1) and fit the a_i and δ_i so as to reproduce the results from the parameterization employed in E791 with the values for the parameters given by their fit with the σ . As in ref.[11] the magnitude and phase of the $\rho(770)$ vector resonance parameters, a_n and δ_n , respectively, are fixed. In that fit apart from the a_i and δ_i , the authors also leave as free parameters the mass and width of the σ . Notice that we

do not include any Blatt-Weisskopf factors in Eq.(2.9). In order to reproduce the results from ref.[11] we fit a Dalitz plot with 20×20 bins (a figure very standard in this E791 analysis) normalized to the total number of events once the background is subtracted, namely 1124 events. This Dalitz plot is generated from the parameterization employed in ref.[11] for the signal only, corresponding to Eq.(2.1). The resulting fit is very good with a low $\chi^2/\nu = 3/152$ and the obtained magnitudes and phases are given in Table 1. On the other hand, we show in Fig.2 the s_{12} projection by the dashed line, while the results from the parameterization of ref.[11] correspond to the points. We then conclude that we are able to reproduce the results of the E791 while keeping the constraint that the phase of the σ contribution follows the $I = 0$ S-wave $\pi\pi$ phase shifts in a rather straightforward manner. An important difference of our fit in Table 1 with respect to the second fit of [11] is the clear dominance of the σ pole in our case, 89%, as compared with the previous reference where its fraction, though also dominant, is lower, around 1/2.

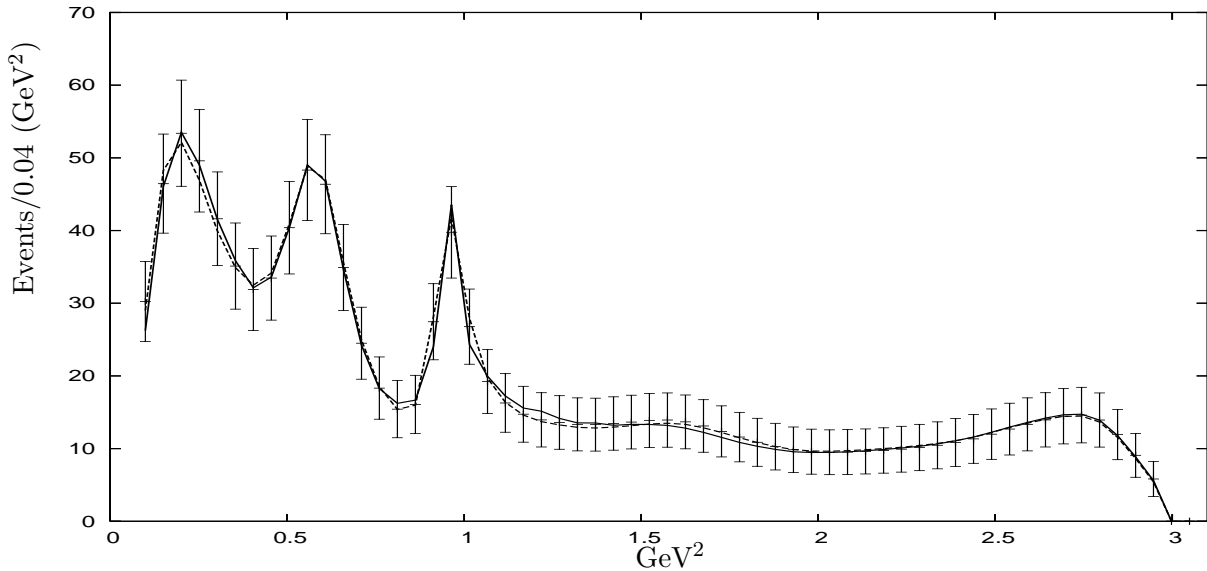


Figure 2: $m^2(\pi\pi)$ projections with "data" points from Eq.(2.1) of ref.[11]. Our results are given by the dashed line with the pole contribution, Eq.(2.1), instead of the BW for the σ of ref.[11]. The solid line corresponds to treat the S-wave FSI following Eq.(2.13).

Now, let us consider the full results of ref.[3]. In this reference the partial waves are written as a product of two matrices (for the general case of coupled channels) as

$$T = [I + N \cdot g]^{-1} \cdot N , \quad (2.10)$$

where $\pi\pi$ is channel 1 and $K\bar{K}$ is channel 2, in such a way that e.g. T_{11} is the $I = 0$ elastic $\pi\pi$ S-wave amplitude and so on. The diagonal matrix $g(s)$ corresponds to the unitarity bubbles of every channel and the matrix N is determined by matching the previous expression with the chiral series at leading order, although the structure of Eq.(2.10) is valid to any order. At lowest order N is the matrix of leading order CHPT amplitudes [3]. Now the important point for us is that given a general vertex N_{ij} , projected in a certain

partial wave and connecting channels i and j , the summation of the unitarity bubbles just implies to multiply the matrix N by the inverse of the matrix $[I + N \cdot g]$, as follows from Eq.(2.10). In such a way that if the final channel ℓ is produced from the initial channel k we then have:

$$T_{\ell k} = \sum_j [I + N \cdot g]_{\ell j}^{-1} N_{jk} , \quad (2.11)$$

so that j runs over all the possible intermediate states, that are also produced from channel k with the original amplitude N_{jk} . Now we want to study a completely analogous situation where channel ℓ is produced from a given production process starting with the D^+ meson. Then, if call by ξ_j the original amplitude for producing channel j the resummation of the unitarity bubbles must be given in exactly the same way as in Eq.(2.11),^{#4} so that:

$$\xi_\ell \rightarrow \sum_j [I + N \cdot g]_{\ell j}^{-1} \xi_j . \quad (2.12)$$

In ref.[3] the $\pi\pi$ and $K\bar{K}$ channels were considered for the study of the meson-meson $I = 0, 1$ S-waves. The σ and $f_0(980)$ appear as poles in the second Riemann sheet of the resulting partial waves. Thus, instead of considering Eq.(2.1), that includes explicitly the BW's associated with these resonances [11], we will instead apply Eq.(2.12) to correct by final state interactions. In this way if we denote by $D = [I + N \cdot g]$, we will have:

$$[D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13})] a_{\pi\pi} e^{i\delta_{\pi\pi}} + [D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13})] a_{K\bar{K}} e^{i\delta_{K\bar{K}}} , \quad (2.13)$$

in return of the contribution for the σ and $f_0(980)$. Notice that we do not include in (2.13) any form factor through Blatt-Weisskopf terms $F_n^{(0)}$ and $F_D^{(0)}$ for the two scalar resonances. The rest of contributions, NR plus the ones from the $f_0(1370)$ and vector and tensor resonances in Table 1 are the same as in ref.[11], although their associated parameters a_n and δ_n are fitted again. As in the case before, we fit a Dalitz plot with 20×20 bins generated from ref.[11] and normalized to the same number of total events. The resulting $\chi^2/\nu = 2/152$ is of the same good quality as before, indicating an accurate reproduction of the amplitudes of ref.[11]. In Table 2 we show the values of the parameters that we have fitted and in Fig.2 we give by the solid line the energy projections of any neutral $\pi\pi$ subsystem.

As a result we see that we are able to reproduce the signal function of ref.[11] rather accurately and at the same time being able to establish that the scattering amplitudes which drive the final state interactions corrections in $D^+ \rightarrow \pi^- \pi^+ \pi^+$ are the same as those determined from scattering data and other production processes. The same conclusion is obtained in ref.[26]. This reference performs a K-matrix fit to data from D^+ and D_s^+ decays to 3π , although the resulting fits have significative lower confidence levels than those of the E791 Collaboration that we reproduce. Regarding the σ resonance we see that it exhibits the same behaviour in the D-decays as in the rest of known processes, although in ref.[26] is excluded. It is worth mentioning that the K-matrix employed in

^{#4}The analytical properties of N_{ij} and ξ_j , in general terms, are the same, that is, analytical functions except for the presence of cuts and poles.

Resonance	a_n	δ_n (radians)	Fraction
NR	0.70	-0.43	17%
$(\pi\pi)\pi^+$	0.31	1.27	102%
$(K\bar{K})\pi^+$	0.11	-0.39	6%
$\rho^+(770)\pi^+$	1 (fixed)	0 (fixed)	36%
$f_0(1370)\pi^+$	0.31	1.99	3%
$f_2(1270)\pi^+$	0.77	1.01	21%
$\rho^0(1450)\pi^+$	0.20	5.48	1%
χ^2/ν	2/152		

Table 2: Results of the reproduction of the parameterization of ref.[11] summarized in Eq.(2.1), employing Eq.(2.13). For each intermediate state we list the resulting magnitude a_n in the second column, the relative phase δ_n in radians in the third column and the fraction of this decay mode in the fourth one.

ref.[26] does not meet the chiral requirement of a soft expansion for low energies and, in particular, it does not fulfill the chiral constraints imposed by the chiral power counting, which requires a $\pi\pi$ scattering amplitude to start at order p^2 , instead of order p^0 as in [26]. This rises serious doubts about the applicability of the results from the K-matrix employed in ref.[26] regarding the existence of the σ resonance since meaningful structures are required in order to extrapolate the T-matrix in the energy complex plane away from the physical real axis, where it has been tested.

Before ending this section let us discuss why the σ resonance is more visible in the D decays than in $\pi\pi$ scattering. One important point is the presence of the huge background (those terms proportional to γ_1, γ_2 , etc... in Eq. (2.1)) in the $I = 0$ S-wave $\pi\pi$ partial wave as discussed at the beginning of this section. There we found out that this background is required in order to preserve the Adler zero in the presence of a light σ resonance, so that a cancellation can occur close to threshold. In this way, the standard pole like structure of a resonance, e.g. that for the case the ρ resonance, is completely destroyed. The question now is why this background does not happen in D decays (or as well in B decays [24]). The main point was already discussed in the second entry of ref.[24], and arguments in this direction were also put forward in ref.[27]. The D or B mesons can be identified with pseudoscalar sources coupled directly to a pion while the other two pions can be thought to couple just to a scalar source. As a direct application of CHPT power counting one realizes that this is not suppressed by any power of momentum or quark mass and then there is a priori no reason why the σ meson should be screened by such large backgrounds as occur in the scattering case. Indeed, this is a result from Eq.(2.13), and can be seen by just making a Laurent expansion of the D^{-1} matrix around the σ pole and then we check the absence of a significant background, contrarily to the scattering case. Certainly the scalar form factor of two pseudoscalars is not renormalization group invariant but this just amounts to a global quark mass multiplying factor and does not induce any energy dependence that could distort the pole structure of the resonance, contrarily to scattering

where the Adler zero happens for a specific energy.

3 FSI in the $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ Decay

In this section we consider the decay $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ where the $f_0(980)$ resonance plays a central role as clearly seen in Fig.3. The "data" points correspond to the signal function of the E791 Collaboration given by the analogous formula to Eq.(2.1) but applied to this other decay. The set of intermediate resonances considered in ref.[16] contains the $f_0(980)$, $\rho^0(770)$, $f_2(1270)$, $f_0(1370)$ and the $\rho^0(1450)$. In the fit performed in ref.[16] the mass and couplings of the $f_0(980)$ were allowed to float with an energy dependent width given by:

$$\Gamma_{f_0}(s) = g_\pi \sqrt{s/4 - m_\pi^2} + g_K \frac{1}{2} \left(\sqrt{s/4 - m_{K^+}^2} + \sqrt{s/4 - m_{K^0}^2} \right), \quad (3.1)$$

to be substituted in the expression for $BW_n(s_{12})$ in Eq.(2.4). The striking fact is the small value for the coupling g_K in the fit of ref.[16], compatible with zero, whereas the coupling of g_π is much larger. This result is very puzzled because is well known that the $f_0(980)$ has great affinity to couple with strangeness sources, as seen e.g. in ϕ decays [22] or in the $J\Psi \rightarrow \phi \pi \pi$ decay [23]. We show that we can reproduce the amplitude for the signal of ref.[16] making use of the amplitudes of ref.[3], where the $f_0(980)$ has more standard properties regarding its couplings. We employ Eq.(2.1) of ref.[16] but, as in the previous section, we remove the $f_0(980)$ BW contribution of [16] and take into account the FSI by employing Eq.(2.13), in terms of the $I = 0$ S-wave $\pi\pi$ and $K\bar{K}$ states. In order to reproduce the amplitudes of ref.[16] we proceed as in section 2. For that we fit a Dalitz plot with 20×20 bins, normalized to 625 events. The Dalitz plot is generated from Eq.(2.1) with the parameters given in ref.[16]. We have performed two types of fits, given in Table 3. One in which the mass and width of the $f_0(1370)$ is fixed to the values of ref.[16] and another in which, as in the previous reference, we let to float their values. The former corresponds to the second and third columns of Table 3 and the latter to the last two columns. Furthermore, in Fig.3 we show the energy projection of any neutral $\pi\pi$ subsystem, the solid line refers to the former fit and the dashed line to the latter.

We see that the reproduction of the signal function of ref.[16] is very fair in both cases. In connection with the fore mentioned unexpected values of the couplings of the $f_0(980)$ as determined in ref.[16], we refer the interested reader to ref.[9] for a detailed study of the masses and couplings of the lightest scalar resonances (σ , κ , $f_0(980)$ and $a_0(980)$), showing that they obey rather accurately a standard SU(3) analysis and constitute the lightest scalar nonet. Other interesting studies on the $f_0(980)$ in D_s and D decays are refs.[28, 29].

Resonance	a_n	δ_n	a_n	δ_n
	Fraction	(radians)	Fraction(%)	(radians)
NR	0.40	0.16	0.40	-0.24
	13%		14%	
$(\pi\pi)\pi^+$	0.28	2.36	0.25	2.23
	6%		5%	
$(K\bar{K})\pi^+$	1(fixed)	0(fixed)	1(fixed)	0(fixed)
	78%		84%	
$\rho^+(770)\pi^+$	0.24	0.14	0.22	0.19
	4%		4%	
$f_0(1370)\pi^+$	0.60	1.68	0.57	2.01
	28%		28%	
$f_2(1270)\pi^+$	0.50	0.39	0.48	0.41
	20%		19%	
$\rho^0(1450)\pi^+$	0.25	0.67	0.24	0.87
	5%		5%	
χ^2/ν	11/142		8.5/140	

Table 3: Results of the reproduction of the parameterization of ref.[16] summarized in Eq.(2.1) for the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$. For each intermediate state we list the resulting magnitude a_n in the second column, the relative phase δ_n in radians in the third column and the fraction of this decay mode just below the value of every a_n , analogously for the fourth and fifth columns. The second and third column correspond to the fit with the mass and width of the $f_0(1370)$ fixed to the values of ref.[16], while in the fourth and fifth columns these values are allowed to float in the fit. In the latter case, $m_{f_0(1370)} = 1.46$ and $\Gamma_{f_0(1370)} = 0.16$ GeV.

4 FSI in the $D^+ \rightarrow K^-\pi^+\pi^+$ Decay

The study of the high statistics Dalitz plot of the decay $D^+ \rightarrow K^-\pi^+\pi^+$, with a sample of 15090 and an estimated background of a 6%, was performed in ref.[12] within the E791 Collaboration in Fermilab. In order to introduce the controversial situation regarding the amplitude employed to describe data in ref.[12], let us briefly discussed the fitting process followed in this reference. When in this reference the amplitude, following the isobar model, is written as the coherent sum of well established resonances, as so quoted in the Particle Data Group ref.[20], the description of the fit is poor. Indeed the χ^2 per degree of freedom found in ref.[12] is $\chi^2/\nu = 167/63$. The main discrepancies between data and the parametrized amplitude come, particularly, from the low energy region, below 0.6 GeV², and at higher energies at around 2.5 GeV². In order to improve the quality of the fit of the Dalitz plot the authors of ref.[12] allowed the mass and width of the scalar resonance $K_0^*(1430)$ to float in the fit and also included Gaussian scalar form factors a la Tornqvist, with the meson radii as additional free parameters. Despite that the χ^2/ν is also around 2. Finally, the authors of ref.[12], together with the previous additions, also included

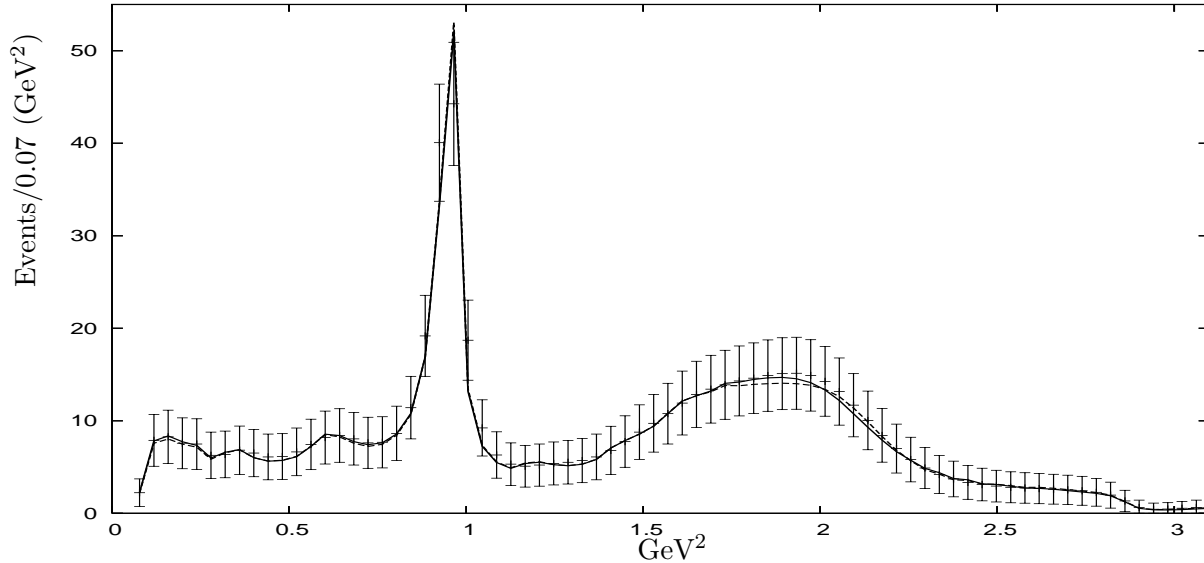


Figure 3: $m^2(\pi\pi)$ projections for "data" [16] and our results, solid and dashed lines, with the FSI given by Eq.(2.12). In the dashed line the mass and width of the $f_0(1370)$ are allowed to float in the fit, second and third columns of Table 3. In the solid line the mass and width of the $f_0(1370)$ are fixed to the values of ref.[16], fourth and fifth columns of Table 3.

the κ resonance, referred as $K_0^*(800)$ in the last edition of the PDG [20] but qualified as controversial. The quality of the fit substantially improves and the $\chi^2/\nu = 46/63$. However, the situation is far from being resolved since: i) the width of the well known $K_0^*(1430)$, allowed to float as discussed above, turned out to be a factor of two lower than its clearly determined value in scattering experiments and other production processes. ii) The Breit-Wigner employed to describe the κ resonance does not follow the elastic S-wave $I = 1/2$ $K\pi$ phase shifts. This is shown in Fig.4 where the points are the elastic S-wave $I = 1/2$ $K\pi$ phase shifts from [30] and the dashed line corresponds to the phase of the relativistic BW of the κ . The discrepancy is manifest, specially close to threshold where the variation for the phase of the BW is much faster than that corresponding to data. Let us remember that the S-wave $I = 1/2$ $K\pi$ scattering is elastic below around 1.3 GeV, since both the $K\eta$ and $K\pi\pi$ channels are negligible below that energy, as clearly shown both experimentally [30, 31] and theoretically [8], and then only the $K\pi$ elastic channel matters in this energy region and the discrepancy cannot come due to inelastic effects.

Denoting by 1 the K^- and by 2 and 3 the equal pions, the amplitudes employed in ref.[12] can be written as:

$$\mathcal{A} = a_0 e^{i\delta_0} \mathcal{N}_0 + \sum_{n=1}^N a_n e^{i\delta_n} \mathcal{A}_n(s_{12}, s_{13}) \mathcal{N}_n. \quad (4.1)$$

The different ingredients contained in the previous equation were discussed in detail in section 2. Here we only mention that in ref.[12] the BW expression (2.4) has opposite sign.

The set of resonances that exchange in Eq.(4.1) contains the $K_0^*(892)$, $K_0^*(1430)$, $K_2^*(1430)$, $K^*(1680)$ and, in the parameterization that reproduces faithfully the experimental data, the κ or $K_0^*(800)$. As discussed above the most controversial aspects present in Eq.(4.1) involves the $I = 1/2$ S-wave $K\pi$ partial wave.

Now, we want to show that we are able to reproduce the amplitude given in Eq.(4.1) as employed in ref.[12] but using the S-wave $I = 1/2$ $K\pi$, $K\eta$ and $K\eta'$ coupled channel partial waves derived in ref.[8]. These T-matrices are obtained from Chiral Perturbation Theory (CHPT) [1, 2] at next-to-leading order, supplemented with the exchange of explicit resonance fields in a chiral invariant manner [32], plus a unitarization scheme compatible with the chiral expansion (chiral unitary approach). Furthermore, additional constraints from large N_c QCD are considered as well in order to restrict the number of free parameters, for further details we refer to refs.[8, 33]. These T-matrices are able to provide an accurate reproduction of the $K\pi$ S-wave amplitude and of the $I = 1/2$ and $I = 3/2$ S-wave phase shifts [30, 31] up to around 2 GeV. Later on, they were employed for calculating through dispersion relations the strangeness changing scalar form factors [33], the light quark masses within QCD scalar sum rules [34] and a crucial counterterm needed in the present and precise studies of $K_{\ell 3}$ decays [35].

The S-wave $I = 1/2$ T-matrices from ref.[8] contains three poles in the appropriate Riemann sheets corresponding to the κ , $K_0^*(1430)$ and the $K_0^*(1950)$. Their pole positions are around $708 - i 305$, $1450 - i 142$ and $1910 - i 27$ MeV, respectively.^{#5} Let us concentrate on the first two resonances which are those that are relevant for the D^+ decay into $K^-\pi^+\pi^+$, since the maximum value of the total center of mass energy of the subsystems (12) or (13) is $m_{D^+} - m_{\pi^+} = 1.73$ GeV, clearly below the influence of the $K_0^*(1950)$ pole. Let us remark that the given pole positions are obtained as a result of the full T-matrices derived in ref.[12], which are not just any sum of pole contributions, although these can dominate in some energy regions. The value for the width of the well established and clearly seen $K_0^*(1430)$ resonance is perfectly compatible with that quoted by the PDG [20], 294 ± 23 MeV,^{#6} while that from the fit of the E791 Collaboration [12], $175 \pm 12 \pm 12$ MeV, is almost a factor of two lower. In addition, the mass obtained in ref.[12] for the $K_0^*(1430)$ is out of the range given in the PDG as well. We also mention that the width of the κ given by the E791 Collaboration, around 400 MeV, is substantially lower than the value from the pole position from ref.[8], around 600 MeV.

Due to the already mentioned discrepancy between the phase motion in energy of the BW for the κ from the E791 Collaboration, let us perform a Laurent series around the κ pole position of the $I = 1/2$ $K\pi$ S-wave amplitude,

$$t_{11} = \frac{\gamma_0^2}{s - s_\kappa} + \gamma_1 + \gamma_2(s - s_\kappa) + \dots \quad (4.2)$$

^{#5}These pole positions vary slightly depending of the fit taken from ref.[8], although we have presented the values from the so called preferred fit, which behaves in the most appropriate way when considering scalar form factors and QCD sum rules.

^{#6}Remember that the *double* of minus the imaginary part of the pole position corresponds to the width of a resonance. This is indeed a possible and unambiguous definition of the width of a resonance.

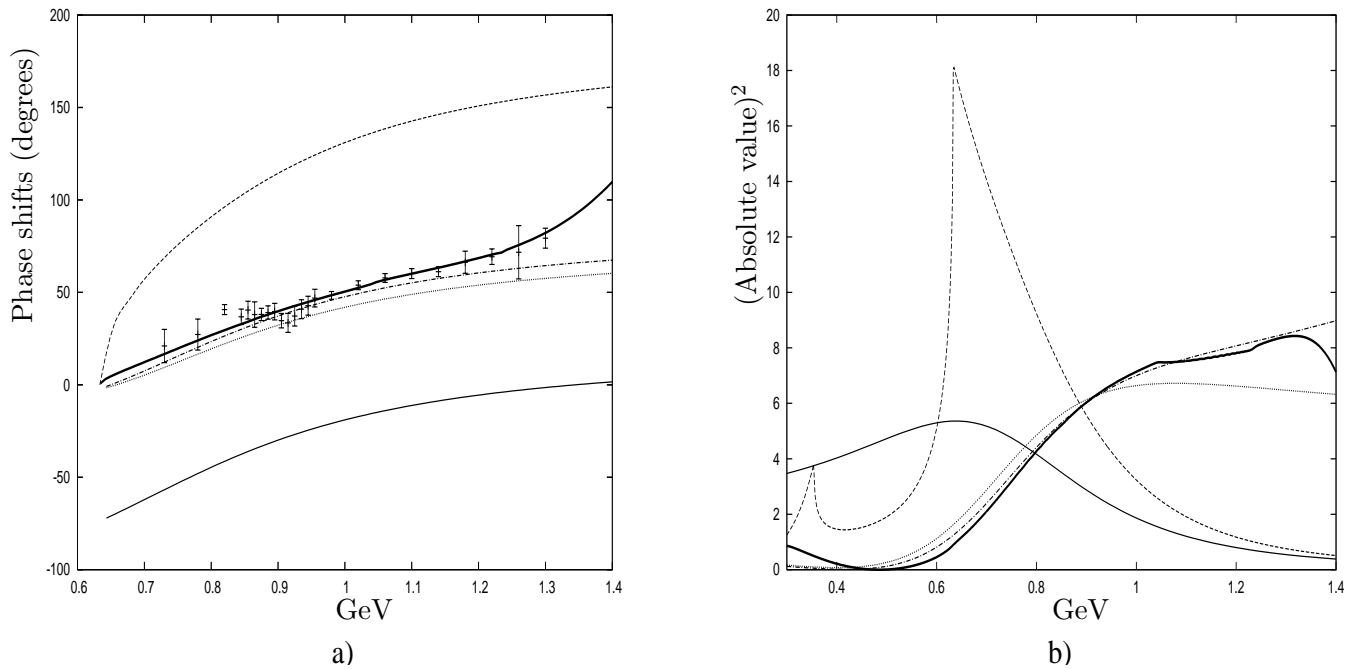


Figure 4: S-wave $I = 1/2$ $K\pi$ phase shifts (left panel) and modulus square of this partial wave, normalized such that the residue at the κ pole is one (right panel). The data points are from ref.[30]. The dashed lines correspond to the BW for the κ resonance employed in ref.[12]. The solid lines correspond to the pure κ pole contribution from Eq.(4.2). The results from the contribution of the κ pole plus the first non-resonant term in Eq.(4.2) are shown by the dotted lines. Finally, the dashed-dotted lines are the results from the Laurent expansion of Eq.(4.2) keeping all the terms shown.

with the pole position and residua obtained from ref.[8], with the values:

$$\begin{aligned} s_\kappa &= (0.71 - i0.31)^2 \text{ GeV}^2, \quad \gamma_0^2 = 19.0 + i5.9 \text{ GeV}^2, \\ \gamma_1 &= 12.5 + i43.1, \quad \gamma_2 = 0.8 + i7.3 \text{ GeV}^{-2}. \end{aligned} \quad (4.3)$$

In Fig.4 we show in the left and right panels the phase and normalized absolute value of this partial wave, respectively. The thinner solid lines correspond to the pure pole contribution in Eq.(4.2). We see in Fig.4a that the phase of this pole contribution does not vanish at threshold, but runs parallel to the experimental phase shifts, so that the difference with respect to them keeps constant along energy up to the increase in the last points due to the closeness of the $K_0^*(1430)$ and the opening of the $K\eta'$ channel. Thus the phase of the pure pole contribution from Eq.(4.2) does follow the motion of the experimental S-wave $I = 1/2$ $K\pi$ phase shifts, in contrast with the BW phase indicated by the dashed line. It is also worth realizing that the phase of the pure κ pole contribution starts at -90 degrees at threshold and this is the reason why its value is not $+90$ degrees at the mass of the κ resonance, 708 MeV, but happens much later. By the same reason, for $s \rightarrow \infty$ one gets only $+90$ degrees from this pole contribution. The agreement between the experimental phase shifts and the expansion (4.2) is reached rather fast. The dotted

lines correspond to keep as well the γ_1 term while the dashed-dotted ones correspond to keep all the terms shown in this equation. The thick solid lines are the full results from ref.[8]. In Fig.4b we consider the absolute value of the partial wave under consideration, but normalized such that the residue at the pole position is one, so that we divide by γ_0^2 the series in Eq.(4.2). The source of each line is the same as already explained and let us notice that this figure starts at around 0.3 GeV, below threshold. We want to stress three important facts: i) The very different behaviour of the κ BW employed by the E791 Collaboration and the absolute value from the partial wave of ref.[8]. ii) The first peak in the BW. This peaks results because the BW formula (2.4) for the κ resonance generates an unphysical pole in the physical sheet below threshold at $0.46 \pm i0.11$ GeV, although it has a negligible impact above threshold. iii) The non-resonant contributions proportional to γ_1 and γ_2 in Eq.(4.2) are as big as the pole contribution, and we see that the final shape of the absolute value of the amplitude is completely distorted as compared with the pure pole contribution. Indeed they almost cancel each other at around 0.48 GeV giving rise to a zero in the full amplitude, as clearly seen in the figure. This is an Adler zero due to chiral symmetry, so that in the chiral limit the pseudoscalar interactions vanish at $s = 0$. At leading order in CHPT [2], this Adler zero sits at 0.48 GeV, very close to the position of the dip in the dotted and dashed-dotted lines and even closer to the zero in the total amplitude which also occurs at around 0.48 GeV. In fact, because of the presence of this Adler zero one can understand why the background turns out to be so big, in the same way as explained in section 2 for the σ case. That is, if there is a pole that largely affects the low energy region, it is then necessary a large background to cancel the pole contribution so that the Adler zero can occur.

Before making use of the full results of ref.[8] let us substitute in Eq.(4.1) the BW contribution given in Eq.(2.4), as employed in ref.[12], by the κ pole contribution,

$$\frac{a_1 e^{i\delta_1}}{s - s_\kappa} , \quad (4.4)$$

located in the position given in Eq.(4.3). This pole contribution, as discussed above and shown in Fig.4a, has a phase motion in agreement with that from the S-wave $I = 1/2$ phase shifts. We keep the rest of terms in Eq.(4.1) and fit the a_i and δ_i so as to reproduce the results from the parameterization employed in E791, Eq.(4.1), with the values for the parameters given by their fit C. As in ref.[12], the magnitude and phase of the $K_0^*(892)$ vector resonance parameters, a_n and δ_n , are fixed to 1 and 0, respectively. In that fit, apart from the a_i and δ_i , the authors also leave as free parameters the masses and widths of the κ and $K_0^*(1430)$, as well as the meson radii that appear in the form factors. Notice that we do not include either any Blatt-Weisskopf factors nor Gaussian form factors in Eq.(4.4). In order to reproduce the results from ref.[12] we fit a Dalitz plot with 20×20 bins normalized to the total number of events once the background is subtracted, namely 28369 events (the charge conjugate decay is also included). This Dalitz plot is generated from the parameterization employed in ref.[11] for the signal function, corresponding to Eq.(4.1). The resulting fit is very good with a low $\chi^2/\nu = 6.5/132$. The values of the resulting magnitudes and phases are given in Table 1. On the other hand, we show in Fig.5 the s_{12} projection by the solid line, while the results from the parameterization of ref.[12]

correspond to the points. The dashed line is the so called high-projection ($s_{12} > s_{13}$) and the dotted line corresponds to the low projection ($s_{12} < s_{13}$). We then conclude that we are able to reproduce the results of the E791 at the same time that the phase of the κ contribution follows the $I = 1/2$ S-wave $K\pi$ phase shifts.

Resonance	a_n	δ_n (radians)	Fraction
NR	2.10	-5.95	53.3%
$\kappa\pi^+$	0.30	-0.63	56.7%
$K_0^*(1430)\pi^+$	1.00	0.92	12.2%
$K_1^*(892)\pi^+$	1 (fixed)	0 (fixed)	12.1%
$K_2^*(1430)\pi^+$	0.17	-0.76	0.4%
$K_1^*(1680)\pi^+$	0.45	0.58	2.5%
χ^2/ν	6.5/132		

Table 4: Results of the reproduction of the parameterization of ref.[12] summarized in Eq.(4.1), removing the BW of the κ by its pole contribution, Eq.(4.4). For each resonance we list the resulting magnitude a_n in the second column, the relative phase δ_n in radians in the third column and the fraction of this decay mode in the fourth one.

We notice that in Table 4 the so called NR contribution plays a more important role, it has a fraction of around 50% while in fit C of the E791 Collaboration amounts to just a 13%.

Now, let us consider the full results of ref.[8] in order to take into account the FSI employing Eq.(2.13). In ref.[8] the partial waves are written as a product of two matrices (for the general case of coupled channel) as $T = [I + N \cdot g]^{-1} \cdot N$ such that e.g. T_{11} is the $I = 1/2$ elastic $K\pi$ S-wave. The diagonal $g(s)$ matrix corresponds to the unitarity bubbles of each channel and the matrix N is determined by matching the previous expression with the chiral series of CHPT plus resonances in the $U(3)$ case, for more details on this respect see ref.[8]. In this reference the $K\pi$, $K\eta$ and $K\eta'$ channels are considered, hence we rewrite Eq.(2.13) for this particular case as,

$$\begin{aligned} & [D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13})] a_{K\pi} e^{i\delta_{K\pi}} + [D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13})] a_{K\eta} e^{i\delta_{K\eta}} \\ & + [D_{13}^{-1}(s_{12}) + D_{13}^{-1}(s_{13})] a_{K\eta'} e^{i\delta_{K\eta'}} . \end{aligned} \quad (4.5)$$

Let us stress that, since the κ and $K_0^*(1430)$ appear as poles in the second Riemann sheet of the partial waves of ref.[8], they are already taken into account when considering Eq.(4.5). Notice that we do not include in Eq.(4.5) for the two scalar resonances either any form factor as done in Eq.(4.1) through the Blatt-Weisskopf terms $F_n^{(0)}$ and $F_D^{(0)}$ nor Gaussian form factors. Furthermore, the $K_0^*(1430)$ resonance from ref.[8] has a mass and a width in complete agreement with the values in the PDG [20], which are clearly determined from the study of $K\pi$ scattering [30, 31]. The rest of contributions present in Eq.(4.1), namely, the NR plus the ones from the vector and tensor resonances in Table 4 are kept, although the parameters a_n and δ_n are fit again so as to reproduce the results

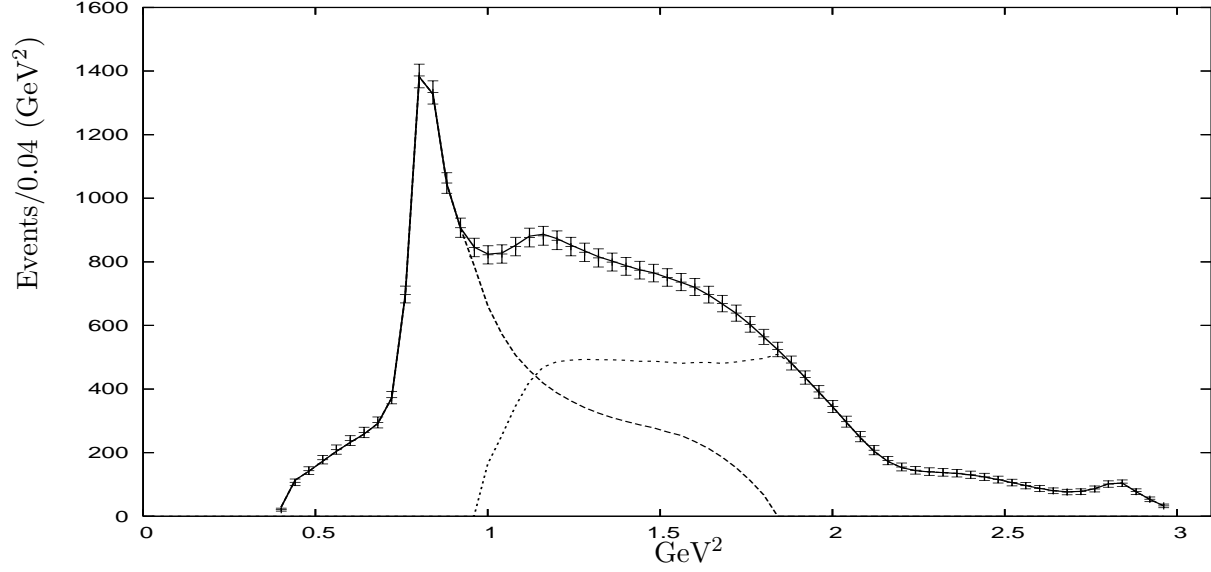


Figure 5: $m^2(K\pi)$ projections for "data" [12] and our results with the pole contribution in Eq.(4.1) instead of the BW for the κ of ref.[12]. The solid, dashed and dotted lines correspond to our results for the projections $m^2(K\pi)$, $m^2(K\pi)_{low}$ and $m^2(K\pi)_{high}$, in order.

of the signal parameterization of the E791 Collaboration. As in the cases before, we fit a Dalitz plot with 20×20 bins generated from ref.[12] and normalized to the total number of events. The resulting $\chi^2/\nu = 127/128$ is acceptable, indicating a rather good reproduction of the amplitudes of ref.[12]. In Table 5 we show the values of the parameters that we have fitted and in Fig.6 we show the energy projections of any neutral $K\pi$ subsystem.

Resonance	a_n	δ_n (radians)	Fraction
NR	1.60	0.10	29.6%
$(K\pi)\pi^+$	1.66	4.10	31.8%
$(K\eta)\pi^+$	0.86	2.63	2.0%
$(K\eta')\pi^+$	2.33	-1.54	9.8%
$K_1^*(892)\pi^+$	1 (fixed)	0 (fixed)	11.6%
$K_2^*(1430)\pi^+$	0.11	-0.62	0.2%
$K_1^*(1680)\pi^+$	0.72	0.80	5.9%
χ^2/ν	127/128		

Table 5: Results of the reproduction of the parameterization of ref.[12] summarized in Eq.(4.1), employing Eq.(4.5). For each intermediate state we list the resulting magnitude a_n in the second column, the relative phase δ_n in radians in the third column and the fraction of this decay mode in the fourth one. For this fit we also leave as free parameters the poorly measured mass and width of the $K_1^*(1680)$, their values are given in the text.

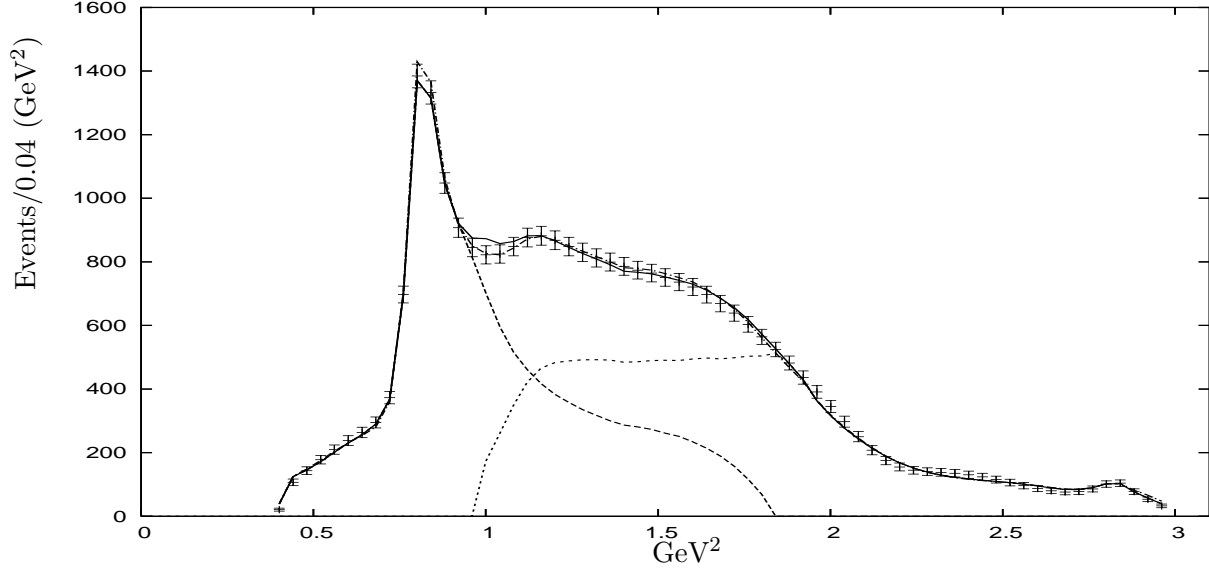


Figure 6: $m^2(K\pi)$ projections for "data" [12] and our results taking into account Eq.(4.5). The solid, dashed and dotted lines correspond to our results for the projections $m^2(K\pi)$, $m^2(K\pi)_{low}$ and $m^2(K\pi)_{high}$, in order. The dashed-dotted line refers to the fit when the $(K\eta)\pi^+$ channel is excluded, Eq.(4.6).

As a result we see that we are able to reproduce the signal function of ref.[12] rather accurately and, at the same time, we are able to establish that the scattering amplitudes driving the final state interactions corrections in $D^+ \rightarrow K^-\pi^+\pi^+$ are compatible with those determined from scattering data [30, 31]. In particular, the κ resonance is present in both and the mass and width of the $K_0^*(1430)$ does neither differ in both cases. On the other hand in the previous fit we have allowed to float the mass and width of the $K_1^*(1680)$, since they are poorly measured. In the PDG [20] the values reported are between 1.7 to 1.8 MeV for the mass and from 0.17 to 0.40 GeV for the width. The values that we obtain in the fit to the parameterization of ref.[12], which employs the central values given in the PDG, are $m = 1.7$ GeV and width $\Gamma = 0.17$ GeV.

Although we have already included the $K\eta$ channel in the fit of Table 5, since this channel has little effect on the $K\pi$ scattering we then check the stability of this fit by removing the $K\eta$ channel. Then, instead of Eq.(2.13) we now consider,

$$\left[D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13})\right] a_{K\pi} e^{i\delta_{K\pi}} + \left[D_{13}^{-1}(s_{12}) + D_{13}^{-1}(s_{13})\right] a_{K\eta'} e^{i\delta_{K\eta'}} . \quad (4.6)$$

The resulting $\chi^2/\nu = 144/130$ is a bit larger than the previous one, and the values of the parameters in the fit are rather similar so that we refrain from presenting them. Nonetheless, we show the event projection in Fig.6 for this case by the dashed-dotted line. On the other hand, the resulting mass and width of the $K^*(1680)$ are 1.7 GeV and 0.13 GeV, in order. Thus, we conclude that our results are rather stable under the presence or removal of the $K\eta$ channel although the resulting χ^2 is somewhat lower when this channel is considered as well.

Finally, the same reasons advocated at the end of section 2 to explain why the σ meson is clearly visible in D^+ decays to three pions can be also applied here for the κ in the $D^+ \rightarrow \pi^-\pi^+\pi^+$ decay.^{#7}

5 Conclusions

We have considered in detail the FSI of the $D^+ \rightarrow \pi^-\pi^+\pi^+$, $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ and $D^+ \rightarrow K^-\pi^+\pi^+$ driven by the S-waves. The Dalitz plots associated with these decays were studied originally in refs.[11, 16, 12] showing by the first time statistically significant evidences for the existence of the σ and κ resonances. Here we have payed special attention to those aspects still controversial after those works with the aim of improving the theoretical basis of the parameterizations employed in these references. In particular, we have shown that the meson-meson S-waves with $I = 0, 1/2$ that drive the corresponding FSI in the previous decays are compatible with those amplitudes determined from studies of scattering data [3, 4, 8], also tested in many other production processes, e.g. [21, 22, 23, 24]. In particular, we have shown that the phase motion of the low energy FSI in the $D^+ \rightarrow \pi^-\pi^+\pi^+$ and $D^+ \rightarrow K^-\pi^+\pi^+$ decays follows the elastic $I = 0$ $\pi\pi$ and $I = 1/2$ $K\pi$ S-waves phase shifts, in order, and how this is compatible with the presence of the σ and κ resonances, respectively. We have seen that the reason for the disagreement between the phases associated with these resonances in the studies of refs.[11, 12] and the experimental phase shifts is the employment by these authors of Breit-Wigner propagators. Once the BW propagator is substituted by the pure pole contributions from Eqs.(2.7) and (4.2), for the σ and κ , respectively, the agreement is restored. Indeed, we have also shown that these pole contributions are not affected by significant backgrounds in the expansion of D^{-1} , see Eqs.(2.13) and (4.5), while huge destructive backgrounds are present in the scattering. In this way the pole contributions are not distorted in the studied D_s and D decays while they are so in the scattering, as shown in Figs.1 and 4. Furthermore, we have also considered the full results for the S-wave FSI from refs.[3, 8] from Eqs.(2.13) and (4.5). In this way we have considered simultaneously the interplay between the σ and the $f_0(980)$ for $I = 0$ (2.13), and that between the κ and $K_0^*(1430)$ for $I = 1/2$ (4.5). This is also important since in ref.[16] the properties of the couplings of the $f_0(980)$ are astonishing while in ref.[12] the mass of the $K_0^*(1430)$ resonance is out of the range given in the PDG [20] and its width is almost a factor 1/2 smaller. Thus, our results show that one can also understand these FSI with "standard" $f_0(980)$ and $K_0^*(1430)$ properties.

Acknowledgments

^{#7}As explained at the end of section 2, but applied now to our case, if we make a Laurent expansion of D^{-1} in Eq.(4.5) around the κ pole one checks that there is no a significant background and D^{-1} is driven by the κ pole contribution around threshold up to about 1 GeV when the influence of the $K\eta'$ channel and that of the $K_0^*(1430)$ resonance starts. Thus, the κ resonance pole structure is not distorted as happens with scattering.

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References

- [1] S. Weinberg, *Physica* **A96** (1979) 327.
- [2] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158** (1984) 142; *Nucl. Phys.* **B250** (1985) 465.
- [3] J.A. Oller and E. Oset, *Nucl. Phys.* **A620** (1997) 435; (E) *Nucl. Phys.* **A652** (1999) 407.
- [4] J.A. Oller and E. Oset, *Phys. Rev.* **D60** (1999) 074023.
- [5] A. Dobado and J. R. Pelaez, *Phys. Rev.* **D56** (1997) 3057; *Phys. Rev.* **D47** (1993) 4883.
- [6] J.A. Oller, E. Oset and J.R. Peláez, *Phys. Rev. Lett.* **80** (1998) 3452; *Phys. Rev.* **D59** (1999) 074001.
- [7] T. Hannah, *Phys. Rev.* **D60** (1999) 017502.
- [8] M. Jamin, J.A. Oller, A. Pich, *Nucl. Phys.* **B587** (2000) 331.
- [9] J.A. Oller, *Nucl. Phys.* **A727** (2003) 353.
- [10] B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, *Phys. Rept.* **353** (2001) 207.
- [11] E791 Collaboration, E.M. Aitala *et al.*, *Phys. Rev. Lett.* **86** (2002) 770.
- [12] E791 Collaboration, E.M. Aitala *et al.*, *Phys. Rev. Lett.* **89** (2002) 121801.
- [13] CLEO Collaboration, H. Muramatsu *et al.*, *Phys. Rev. Lett.* **89** (2002) 251802.
- [14] Belle Collaboration, K. Abe *et al. et al.*, hep-ex/0308043.
- [15] BaBar Collaboration, B. Aubert *et al.*, hep-ex/0408088.
- [16] E791 Collaboration, E.M. Aitala *et al.*, *Phys. Rev. Lett.* **86** (2001) 765.
- [17] For a not so recent review see J.A. Oller, E. Oset and A. Ramos, *Prog. Part. Nucl. Phys.* **45** (2000) 157.
- [18] J. A. Oller and U. G. Meißner, *Phys. Lett. B* **500** (2001) 263.
- [19] J.A. Oller, *Nucl. Phys.* **A725** (2003) 85.
- [20] S. Eidelman *et al.*, *Phys. Lett.* **B592** (2004) 1.
- [21] J.A. Oller and E. Oset, *Nucl. Phys.* **A629** (1998) 739.

- [22] J.A. Oller, Nucl. Phys. **A714** (2003) 161; Phys. Lett. **B426** (1998) 7; J.E. Palomar, L. Roca, E. Oset and M.J. Vicente Vacas Nucl. Phys. **A729** (2003) 743; E. Marco, S. Hirenzaki, E. Oset and H. Toki Phys. Lett. **B470** (1999) 20.
- [23] U.-G. Meißner and J.A. Oller, Nucl. Phys. **A679** (2001) 671; L. Roca, J.E. Palomar, E. Oset and H.C. Chiang, hep-ph/0405228.
- [24] S. Gardner and U.-G. Meißner, Phys. Rev. **D65** (2002) 094004; J.A. Oller, eConf C0304052:WG412,2003.
- [25] J.R. Peláez, Phys. Rev. Lett. **92** (2004) 102001.
- [26] FOCUS Collaboration, J. M. Link *et al.*, Phys. Lett. B **585** (2004) 200.
- [27] D. Bugg, Phys. Lett. **B572** (2003) 1; (E) Phys. Lett. **B595** (2004) 556.
- [28] I. Bediaga, F.S. Navarra and M. Nielsen, Phys. Lett. **B579** (2004) 59.
- [29] I. Bediaga and M. Nielsen, Phys. Rev. **D68** (2003) 036001.
- [30] P. Estabrooks *et al.*, Nucl. Phys. **B133** (1978) 490.
- [31] D. Aston *et al.*, Nucl.Phys. **B296** (1988) 493.
- [32] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. **B223** (1989) 425; G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311.
- [33] M. Jamin, J.A. Oller, A. Pich, Nucl. Phys. **B622** (2002) 279.
- [34] M. Jamin, J.A. Oller, A. Pich, Eur. Phys. J. **C24** (2002) 237.
- [35] M. Jamin, J. A. Oller and A. Pich, JHEP **0402** (2004) 047.